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that magnitude is taken. He never gave any arithmetical or algebraic proof of the continuity of any number-system.

Austin, Texas.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJ. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc., Curry University, Pittsburg, Pennsylvania.

[Continued from June-July Number.]

II. Proofs Resulting from Straight-line Properties of the Circle.

XV. Let ABC be $\triangle$ right-angled at C. With either extremity, as B, of the hypotenuse, as a center, and with a radius equal to the hypotenuse, describe a circle. Produce the legs of the $\triangle$ to chords. One of the chords, as DE, will be a diameter.

Then $AC.CL=DC.CE$, or $b^2=(c-a)(c+a)$.

$\therefore c^2=a^2+b^2$.

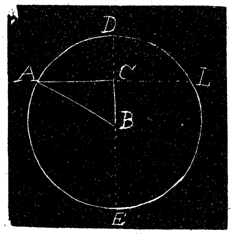

Fig. 11.

XVI. Let ABC be $\triangle$ right-angled at C. With either extremity, as B, of the hypotenuse, as the center, and with a radius equal to the adjacent leg, describe a circle. Produce the hypotenuse to a secant.

Then $\overline{AC}^2=AE.AD$, or $b^2=(c-a)(c+a)$.

$\therefore c^2=a^2+b^2$.

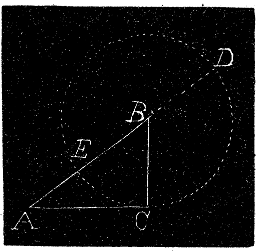

Fig. 12.

Note.—This method is given by Richardson in *Runkle's Mathematical Monthly*, No. II, 1859; also by Hoffmann, and, in a slightly different form, by Wipper, the latter stating that the proof is found in "Huberti Rudimenta Algebræ," Wurceb, 1792. It was known to the writers, however, independently of these sources.

XVII. Let ABC be a $\triangle$ right-angled at C.

Case I. When the two legs are unequal.

With C as a center, and with the shorter leg, as BC, as a radius, describe a circle. Produce AC to a secant. Draw CL perpendicular to AB.

Then $AD.AH=AE.AB$,

or $(b-a)(b+a)=c(c-2LB)$.

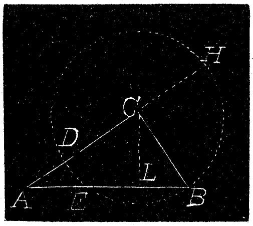

Fig. 13.

Substituting for LB any of its equivalents in terms of the sides of the given $\triangle$, and reducing, we get, $c^2=a^2+b^2$.

Case 2. *When the two legs are equal.*

We easily pass, by the usual method of the theory of limits, from Case 1 to Case 2.

XVIII. Same as in XVII, except that the circle is described with the longer leg, as AC, as a radius. Then, produce all the sides to chords.

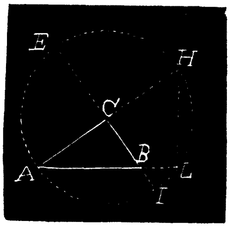

Fig. 14.

Then $AB.BL=BE.BD$,

or $c.BL=(b+a)(b-a)$...... ....(1).

Also, $AB : AH :: AC : AL$,

or. $c : 2b :: b : c+BL$,

whence, $c^2+c.BL=2b^2$ ....(2).

(1) in (2), $c^2=a^2+b^2$.

When the legs are equal, we pass from the case given as suggested in Case 2 of XVII.

XIX. Same as in XVII, except that both cases are treated alike, and the circle is described with a radius equal to the perpendicular from C to AB. Then produce the legs to secants, and draw CD.

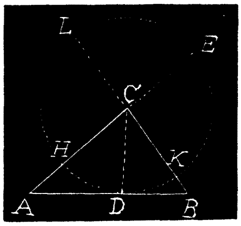

Fig. 15.

Then $\overline{AD}^2=AH.AE=b^2-\overline{CD}^2$;

$\overline{BD}^2=a^2-\overline{CD}^2$;

also, $2AD.DB=2\overline{CD}^2$.

Adding, $c^2=a^2+b^2$.

XX. Let ABC be a △ right-angled at C. Produce either leg, as AC, through C, making $CD=AC$. Join BD. Circumscribe a circle about △ ABD, and produce BC to a diameter.

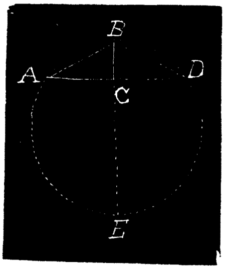

Fig. 16.

Then $\overline{BC}^2=AB.BD-AC.CD$,

or $a^2=c^2-b^2$.

$\therefore c^2=a^2+b^2$.

XXI. Fig. 16.

$AB.BD=BE.BC$,

or $c^2=a^2+a.CE=a^2+b^2$.

Note.—The last two are special cases of familiar propositions, and are given by various writers.

[To be Continued.]